



Mathematics Notes for Revision

General Tips

1 - Check your calculate mode at all times

Questions may require your calculator to be in radian or degree mode. Be sure to double check it before you start attempting angle or trigonometry questions.

2 - Study the formula sheet

One of the most overlooked tips that students make. The formula sheet is the only thing that you have access to outside of the examination. Hence, make full use of it!

3 - Construction questions

Compass arcs showing your construction method are required so do not erase them out – they are an important part of your working.

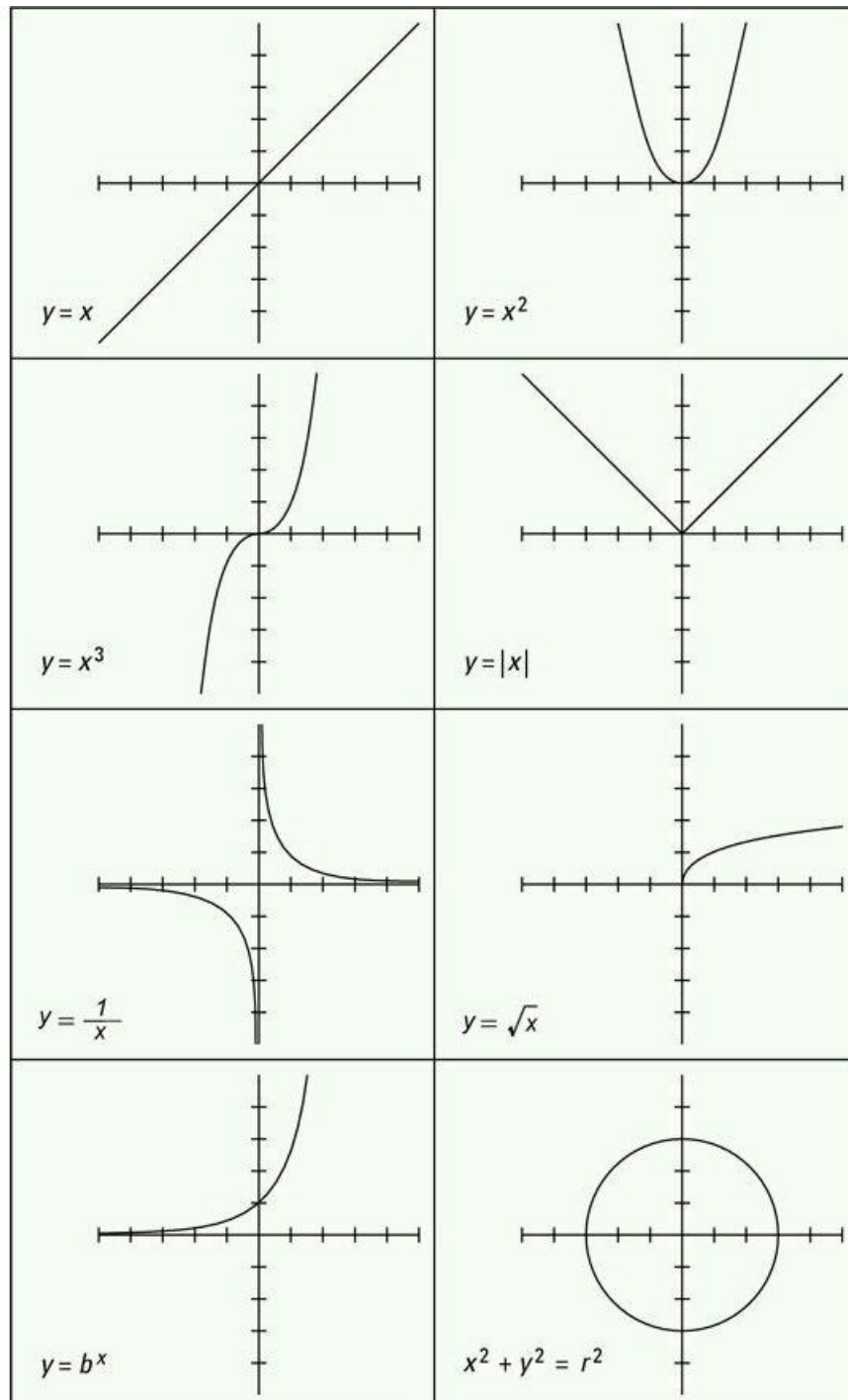
4 - Reading the question

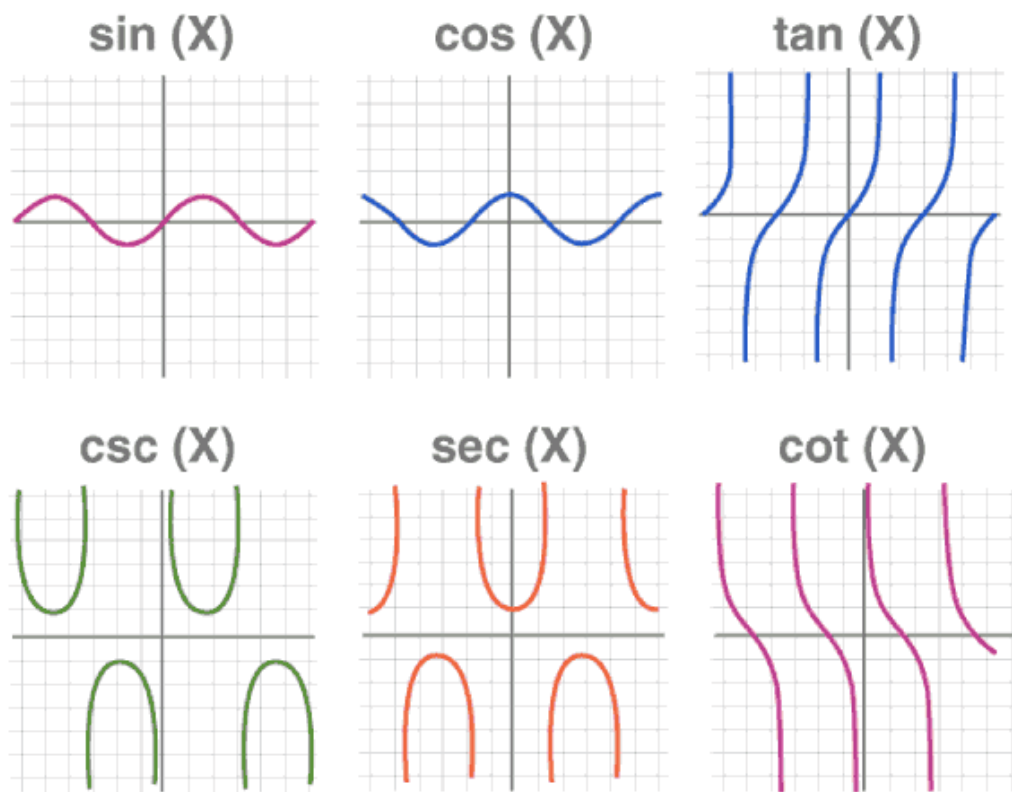
If the question specifically asks you to “show all your workings”, then you are advised to show step by step workings. Marks are given for the work that you do correctly, not subtracted for the work that you get wrong. Even if you do get the final answer wrong, you will be awarded method marks and even error carry forward marks.

5 - Knowing simple conversions

It is advisable to know some simple conversions at the back of your mind, instead of memorising them only. Some conversions that you should know at the tip of your fingers are - km/h to m/s, days to seconds, decimal values of time etc.

1 - Graph Functions

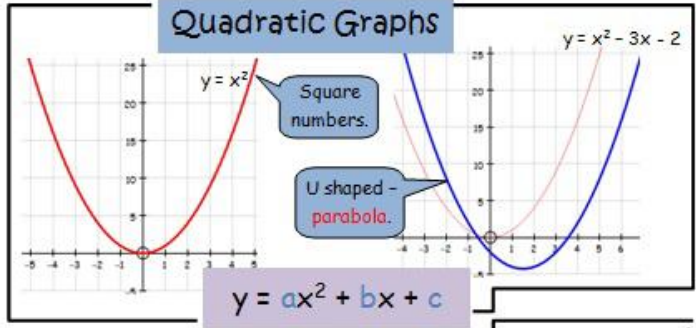
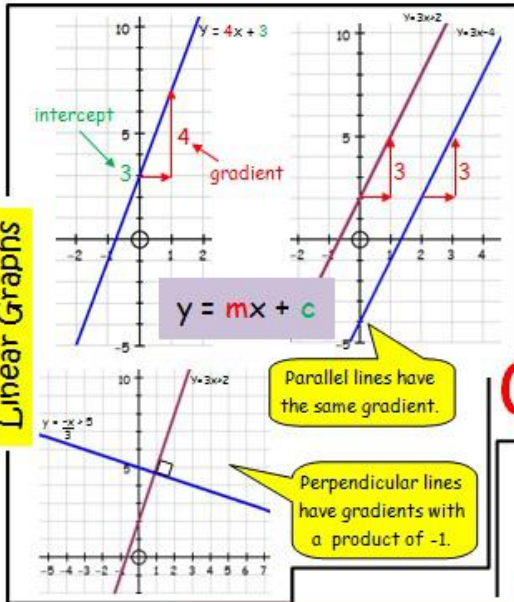




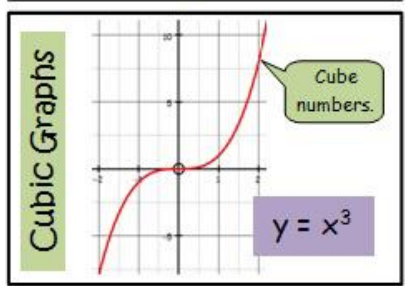
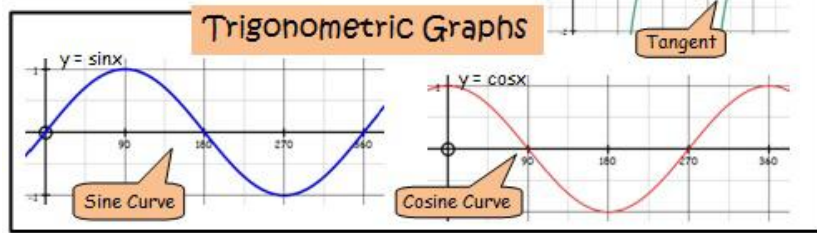
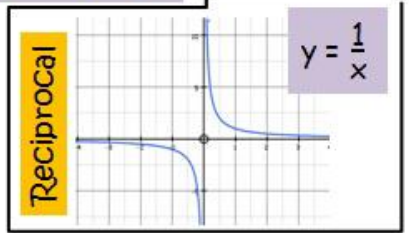
Alternatively, you can visit this graph plotter, where you can type the specific equation, it will generate the graph with respect to the equation.

<https://www.transum.org/Maths/Activity/Graph/Desmos.asp>

Linear Graphs



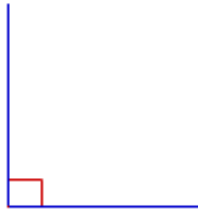
GRAPHS



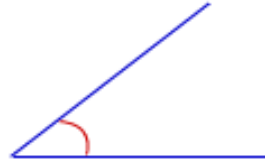
2 - Prefixes

| Prefix | Symbol | Multiplier | |
|---------------------|---------------|------------|---------------------------|
| exa | E | 10^{18} | 1,000,000,000,000,000,000 |
| peta | P | 10^{15} | 1,000,000,000,000,000 |
| tera | T | 10^{12} | 1,000,000,000,000 |
| giga | G | 10^9 | 1,000,000,000 |
| mega | M | 10^6 | 1,000,000 |
| kilo | k | 10^3 | 1,000 |
| hecto | h | 10^2 | 100 |
| deka | da | 10^1 | 10 |
| deci | d | 10^{-1} | 0.1 |
| centi | c | 10^{-2} | 0.01 |
| milli | m | 10^{-3} | 0.001 |
| micro | μ | 10^{-6} | 0.000,001 |
| nano | n | 10^{-9} | 0.000,000,001 |
| pico micro micro | P $\mu\mu$ | 10^{-12} | 0.000,000,000,001 |
| femto | f | 10^{-15} | 0.000,000,000,000,001 |
| atto | a | 10^{-18} | 0.000,000,000,000,000,001 |

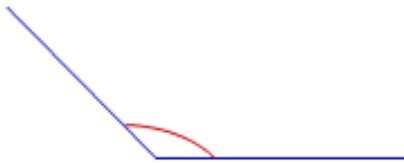
3 - Types of Angles and Triangles



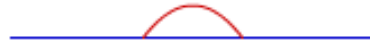
Right Angle, **90°**



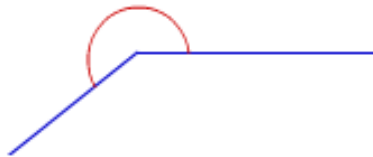
Acute Angle,
Less than **90°**



Obtuse Angle,
Greater than **90°**
Less than **180°**



Straight Angle,
Exactly **180°**



Reflex Angle,
Greater than **180°**
Less than **360°**

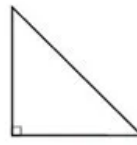


Adjacent Angles,

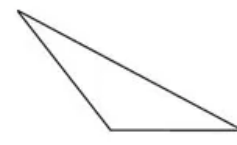
By Angle



Acute
all angles < 90

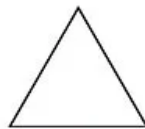


Right
one angle = 90

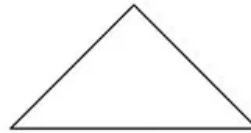


Obtuse
one angle > 90

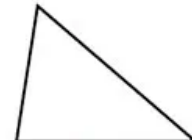
By Side



Equilateral
3 equal sides



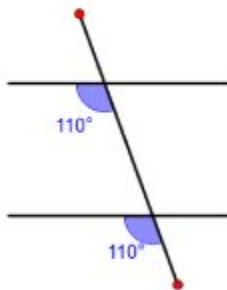
Isosceles
2 equal sides



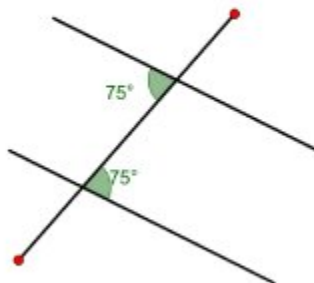
Scalene
no equal sides

Angle Properties

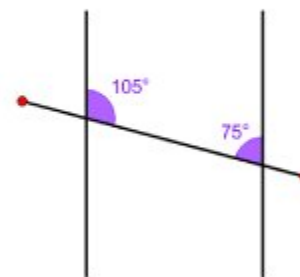
Corresponding Angles



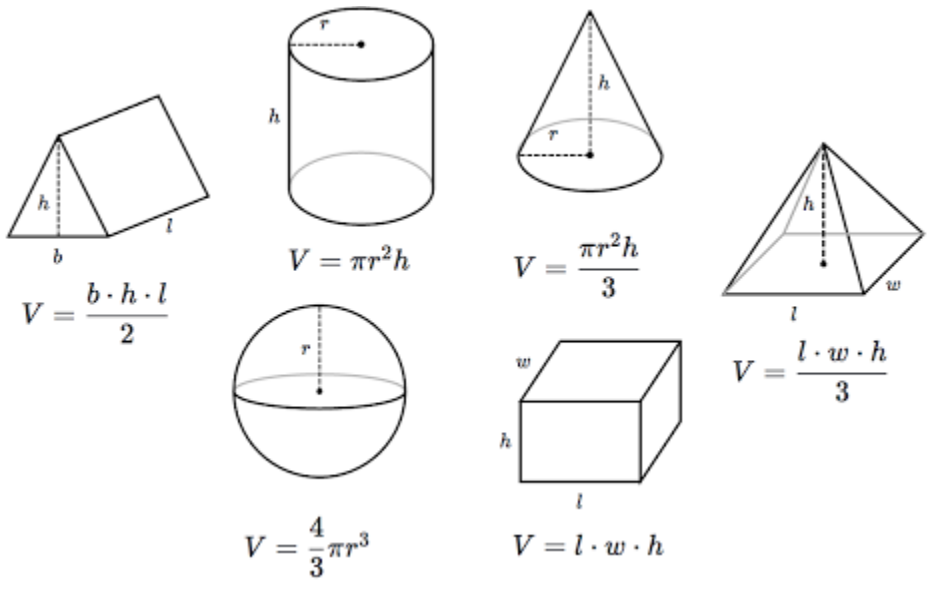
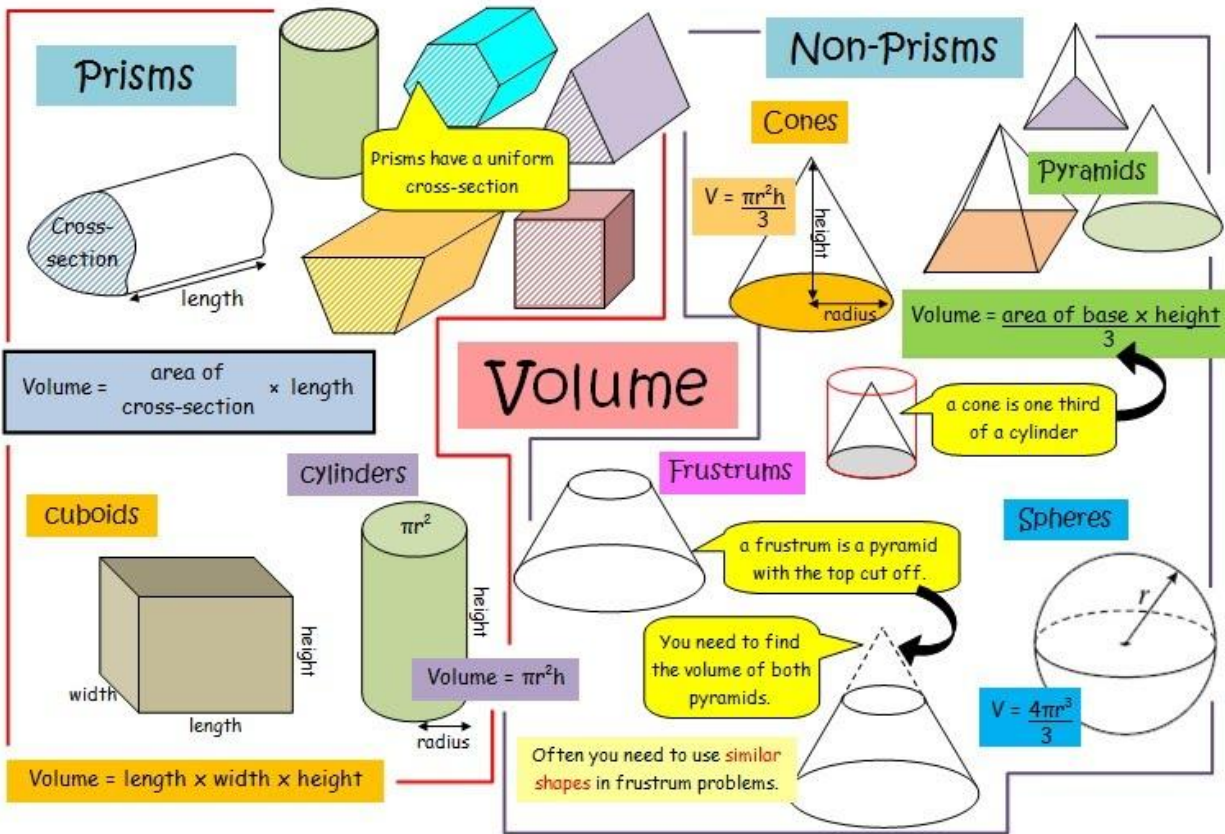
Alternate Angles



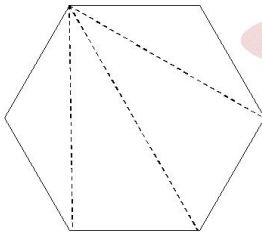
Interior Angles



4 - Volume & Polygons



Angle Sum



$$4 \times 180^\circ = 540^\circ$$

$$(n - 2) \times 180^\circ$$

number of
triangles



3 triangle



4 quadrilateral



5 pentagon



6 hexagon

7 - heptagon



8 octagon

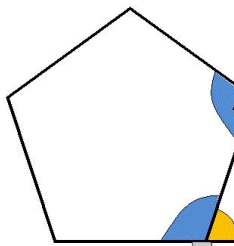
9 - nonagon

10 - decagon

Polygons



interior angle



$$\frac{\text{angle sum}}{\text{number of sides}}$$

OR

$$180^\circ - \text{exterior angle}$$

exterior angle

$$\frac{360^\circ}{\text{number of sides}}$$

OR

$$180^\circ - \text{interior angle}$$

5 - Quadratic Equations

Solving:

- Factorising
- Formula
- Completing the square
- Drawing a graph

Completing the square:

| | |
|-----|-------|
| x | 2 |
| x | x^2 |
| 2 | $2x$ |
| | 4 |

$\Rightarrow x^2 + 4x = (x + 2)^2 - 4$
complete the square

$x^2 + 4x - 3 = 0$ (half of $4x$)
 $(x + 2)^2 - 4 - 3 = 0$
 $(x + 2)^2 - 7 = 0$ (subtract 2^2)
 $x + 2 = \pm\sqrt{7}$
 $x = \pm\sqrt{7} - 2$

Factorising:

$x^2 + 9x + 20 = 0$

| | |
|-----|-------|
| x | 5 |
| x | x^2 |
| 4 | $4x$ |
| | 20 |

fill in the blue squares first
work out the factors (red numbers)

$(x + 5)(x + 4) = 0$
 $x = -5 \quad x = -4$

Quadratic Equations
 $ax^2 + bx + c$

$3x^2 - 10x + 8 = 0$

the white squares make the x term

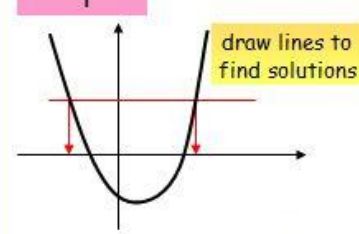
| | |
|------|--------|
| $3x$ | -2 |
| $3x$ | $3x^2$ |
| -4 | $-6x$ |
| | 8 |

$(3x - 4)(x - 2) = 0$
 $x = \frac{4}{3} \quad x = 2$

The formula:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Graphs:



draw lines to find solutions

Parabola - u shaped graph

Difference of Two Squares:

$x^2 - 16$
 $(x - 4)(x + 4)$

x squared subtract 4 squared

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Helpful Expansions to Remember

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

6 - Binomial Expansions

Binomial Expansion

$$\begin{aligned}(a+b)^n &= {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_n a^0 b^n \\ &= a^n + n a^{n-1} b + \frac{n(n-1)}{2!} a^{n-2} b^2 + \frac{n(n-1)(n-2)}{3!} a^{n-3} b^3 + \dots + b^n\end{aligned}$$

$$\begin{aligned}(1+b)^n &= {}^nC_0 b^0 + {}^nC_1 b^1 + {}^nC_2 b^2 + \dots + {}^nC_n b^n \\ &= 1 + nb + \frac{n(n-1)}{2!} b^2 + \frac{n(n-1)(n-2)}{3!} b^3 + \dots + b^n\end{aligned}$$

7 - Partial Fractions Decomposition Formulas

| Factor in denominator | Term in partial fraction decomposition |
|-----------------------|--|
| $ax + b$ | $\frac{A}{ax + b}$ |
| $(ax + b)^k$ | $\frac{A_1}{ax + b} + \frac{A_2}{(ax + b)^2} + \dots + \frac{A_k}{(ax + b)^k}, \quad k = 1, 2, 3, \dots$ |
| $ax^2 + bx + c$ | $\frac{Ax + B}{ax^2 + bx + c}$ |
| $(ax^2 + bx + c)^k$ | $\frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \dots + \frac{A_kx + B_k}{(ax^2 + bx + c)^k}, \quad k = 1, 2, 3, \dots$ |

8 - Percentages

on a calculator

39% of 82
 0.39×82

Change to a decimal and multiply

increasing

Increase £60 by 12%

12% of 60 = $0.12 \times 60 = \text{£}7.20$

New amount = $\text{£}60 + \text{£}7.20 = \text{£}67.20$

ADD

fraction to %

$\frac{15}{20} = \frac{75}{100} = 75\%$ (multiplied by 5)

OR

$15 \div 20 \times 100 = 75\%$

decreasing

decrease £60 by 12%

12% of 60 = $0.12 \times 60 = \text{£}7.20$

New amount = $\text{£}60 - \text{£}7.20 = \text{£}52.80$

SUBTRACT

without a calculator

| | |
|---------------------|--------------------|
| 50% - half | 10% - divide by 10 |
| 25% - half and half | 5% - half 10% |
| 75% - 50% + 25% | 20% - double 10% |

Simple Probability

$$\text{Probability} = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$



Example:

$$P(\text{red}) = \frac{7}{12}$$

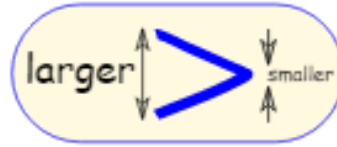
← Number of red marbles
← Total number of marbles (sample space)

$$P(\text{blue}) = \frac{5}{12}$$

← Number of blue marbles
← Total number of marbles (sample space)

10 - Inequalities

Equality and Inequality



$=$ equal

\neq not equal

$>$ greater than

\geq greater than or equal

$<$ less than

\leq less than or equal

11 - Trigonometry Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \qquad \cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \qquad \csc \theta = \frac{1}{\sin \theta}$$

$$\sin(\theta + 2\pi) = \sin \theta$$

$$\cos(\theta + 2\pi) = \cos \theta$$

$$\tan(\theta + \pi) = \tan \theta$$

Trigonometry Hand Trick:

<https://youtu.be/TyrM8G1MqiI>

12 - Simple conversions

Radians and Degrees

$$\text{Radians} = \left(\frac{\pi}{180^\circ} \right) \times \text{degrees}$$

$$\text{Degrees} = \left(\frac{180^\circ}{\pi} \right) \times \text{radians}$$

km/h to m/s and vice versa

$$\frac{\text{m}}{\text{s}} = \frac{\div 1000 \text{ km}}{\div 3600 \text{ hr}}$$

